

Solving Inequalities: Multiplying or Dividing by a Negative Number

We solve inequalities the same way we solve equations, with one exception. When we divide or multiply both sides of an inequality by a *negative* number, the direction of the inequality sign changes. We can see the proof of this in the following examples.

$$15 < 27$$

$$\frac{15}{-3} ? \frac{27}{-3}$$

$$-5 > -9$$

-5 is greater than -9, so we must reverse the sign to make the solution true.

$$2 < 8$$

$$2(-2) ? 8(-2)$$

$$-4 > -16$$

-4 is greater than -16, so we must reverse the sign to make the solution true.

Therefore, when solving an inequality, if you multiply or divide by a negative number, be sure to reverse the inequality sign.

$$-2x + 13 < 27$$

$$\frac{-2x + 13}{-13} < \frac{27}{-13}$$

Subtract 13 from both sides.

$$-2x < 14$$

$$\frac{-2x}{-2} > \frac{14}{-2}$$

Divide both sides by (-2) and reverse the inequality sign.

$$x > -7$$

Solution.

Remember:

- If you must multiply or divide by a negative number, you must also change the direction of the inequality sign.

Solve the inequalities. For some of them, you will need to reverse the inequality sign.

1. a. $-7x < 14$

b. $\frac{y}{4} - 8 > 12$

c. $\frac{z}{-6} + 5 > 3$

Lesson 11

2. a. $\frac{x}{-2} - 9 < 0$

b. $-2y + 6 < 20$

c. $3z + 5 > -4$



We Remember

Complete the table showing the number of different kinds of customers served by Westside Equipment last year. Then construct a rectangle graph of this information.

△ 3.

Kind of Customer	Number Sold	Percent
New Retail	251	
Repeat Retail	399	
Repeat Wholesale	307	
New Wholesale	43	
Total		

Reduce to simplest form.

4. a. $\frac{p^2q^4}{p^4q^2} =$

b. $\frac{4x^5}{2x^4} =$

5. The 94 miles from Bill and Gail's home to Bill's parents' home is 7 times farther than the distance to Gail's parents' home. How far do Bill and Gail live from Gail's parents?

Round to the nearest mile. _____

Simplify the expressions.

6. a. $y^{-3} \div y^{-9}$

b. $x^8 \div x^4$

c. $2a^3 \cdot 7b^2$

d. $4z^4 \cdot 6z^{-1}$

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7. The middle number in an ordered list is the _____.
8. The formula for finding the volume of a pyramid or cone is _____.
9. A number factor in a term used with a variable is known as a numerical _____.
10. a. 1 acre = _____ feet² b. 1 mile² = _____ acres
11. a. 1 gallon \approx _____ liters b. 1 kilogram \approx _____ pounds
12. a. $\frac{3}{8} =$ _____% b. $\sqrt{169} =$ _____ c. $2^4 =$ _____ d. $5^3 =$ _____

Write the number of significant digits in each box. Underline the digit in the answer to show which digit will be rounded. Round the answers according to the fewest number of significant digits.

13. a.
$$\begin{array}{r} 9,527 \\ \times \quad 600 \\ \hline 5,716,200 \end{array} \approx \underline{\hspace{2cm}}$$
- b.
$$\begin{array}{r} 2.7 \\ 4.07 \\ + 1.19 \\ \hline 7.96 \end{array} \approx \underline{\hspace{2cm}}$$
14.
$$\begin{array}{r} 8.\overline{48} \\ \square 66 \overline{)560} \end{array} \approx \underline{\hspace{2cm}}$$

Simplify.

15. a. $\frac{210}{(9+6)(11-4)}$ b. $\frac{40 - (10 + 6)}{40 - 10 + 6}$

Solve.

16. a. $-12b = -50$ b. $-n = 14$ c. $\frac{4}{3}m = 8$ ☆ d. $p(p+2) - 2p = 100$

17. Bill arranged to have equipment hauled in from an auction 475 miles away. A hauler from the auction house delivered the equipment for \$1.90 per mile. How much did it cost to have the equipment delivered? _____

Lesson 11

+ Skill Builders

18. a. $\frac{3 \text{ hr } 40 \text{ min}}{9}$

b. $3\frac{3}{4} \div 5\frac{5}{6} = \underline{\hspace{2cm}}$

c. $\frac{4}{-1\frac{5}{16}}$



Find the mean, median, and mode. Round to the nearest whole.

93, 84, 96, 89, 100, 84, 95, 84, 96, 98, 88

19. a. mean $\underline{\hspace{2cm}}$

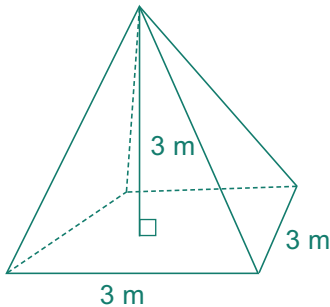
b. median $\underline{\hspace{2cm}}$

c. mode $\underline{\hspace{2cm}}$

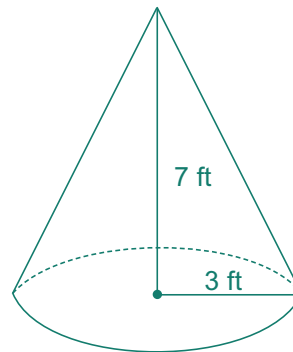


20. The heavy-duty diesel oil that Westside buys for the equipment costs \$15.99 for 2 gallons or \$34.99 for 5 gallons. Which is the better buy? $\underline{\hspace{2cm}}$

Find the volumes. Use $\frac{22}{7}$ for π .



21. a. $\underline{\hspace{2cm}}$



b. $\underline{\hspace{2cm}}$



22. Westside considered making quarterly payments instead of yearly payments for liability insurance. The insurance company would charge an additional 4% for the extra work that quarterly billings would make. Westside presently pays \$6,265 per year. What would the yearly amount be if they were to pay quarterly? $\underline{\hspace{2cm}}$



23. When Bill and Gail visited his parents they traveled 94 miles in $1\frac{3}{4}$ hours. What was their average speed? Use the $d = rt$ formula, and round to the nearest whole. _____



24. One evening, the Weavers rode their bikes six miles on the rail trail and then back again. They averaged 9 miles per hour and rested 15 minutes by the stream where they turned around. If they left at 6:15 p.m., what time did they return?

Change to standard notation.

25. a. $2.02 \times 10^3 =$ _____ b. $6.8 \times 10^{-4} =$ _____

Complete this sentence.

26. Reverse the inequality sign if you _____ or _____ both sides by a negative.

Solve.

27. a. $6a + 9 < -15$ b. $\frac{b}{-3} + 12 < 11$ c. $-5c - 7 > 18$



Sine, Cosine, and Tangent Applications

You can use a trigonometry chart to find missing information about right triangles. When solving story problems, sketch a triangle and fill in the given information to help you decide which trigonometric ratio to use.

Lesson 12

Example 1: LaRay wants to cut rafters for a porch roof he is preparing to frame against the side of a house. The porch is 8 feet wide, and he wants the roof to be 3 feet higher against the house than at the edge of the porch. How many degrees should he cut the angle on the end of each rafter so it will fit snugly against the house?

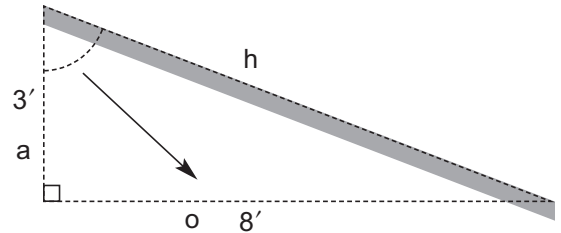
What is the known information about this triangle? *The adjacent side is 3 ft, and the opposite side is 8 ft.*

What is the desired information? Since he knows the lengths of the opposite side and the adjacent side, *LaRay needs to know the tangent so he can figure out what angle to cut the rafter ends.*

Formula: $\tan = \frac{\text{opposite}}{\text{adjacent}}$

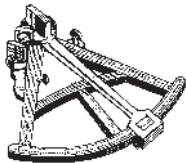
Solve for the tangent: $\tan = \frac{8}{3}$

$$\tan \approx 2.667$$

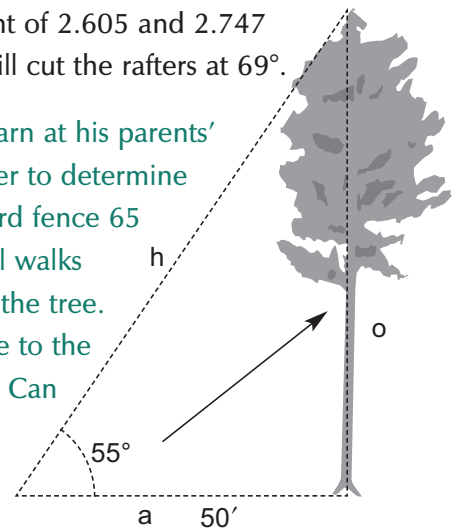


After finding the tangent in Example 1 above, use the trigonometric chart on page 67 to find the closest degree match for the tangent. Since 2.667 is between the tangent of 2.605 and 2.747 and is slightly closer to 2.605, the angle should be 69°. Therefore, he will cut the rafters at 69°.

Example 2: Bill wants to cut down a large pine tree behind the barn at his parents' farm. He needs to know the height of the tree in order to determine whether he can fell the tree without hitting the orchard fence 65 feet downhill from the tree. Carrying an astrolabe, Bill walks down the hill until his eyes are level with the base of the tree. At this point, 50 feet from the tree, he sights the angle to the top of the tree to be 55 degrees. How tall is the tree? Can Bill cut down the tree without taking out the orchard fence?



astrolabe



Known information: *The adjacent side is 50 ft, with a 55° angle.*

Desired information: *What is the length of the opposite side?*

Formula: $\tan = \frac{\text{opposite}}{\text{adjacent}}$

From the chart on page 67, the tangent for 55° is 1.428.

Substitute and solve. $1.428 \approx \frac{\text{opposite}}{50}$

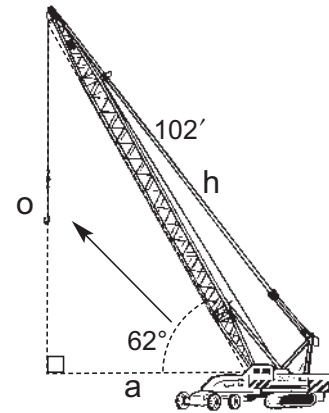
$$1.428 \cdot 50 \approx \frac{\text{opposite}}{50} \cdot 50$$

The tree is about 71 feet tall.

$$71.4 \approx \text{opposite}$$

Since the orchard fence is 65 feet away from the tree and the tree is about 71 feet tall, Bill will have to move the fence before he cuts the tree.

Example 3: Delmar used a crane with a 102' boom to set equipment into the basement of a building under construction. The gauge on the crane boom registered a 62° angle as he lowered a piece into place. What is the horizontal distance between the pivot point of the boom and the cable lowering the equipment?



Known information: *The hypotenuse is 102 ft with a 62° angle.*

Desired information: *What is the length of the adjacent side?*

Formula: $\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$

Cosine for 62° from the chart: 0.469

$$0.469 \approx \frac{\text{adjacent}}{102}$$

Substitute and solve. $0.469 \cdot 102 \approx \frac{\text{adjacent}}{102} \cdot 102$

$$47.838 \approx \text{adjacent}$$

The cable is about 48 feet from the pivot point.

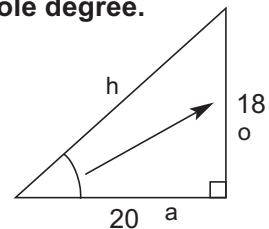
Remember:

- If you know the *opposite* and the *adjacent*, you solve for **tangent**.
- If you know the *opposite* and the *hypotenuse*, you solve for **sine**.
- If you know the *adjacent* and the *hypotenuse*, you solve for **cosine**.



Use the chart on page 67 to find the angle measurement to the nearest whole degree.

1. a. Known information: The lengths of the _____ and the _____ sides
- b. Desired information: The _____ for the marked angle.
- c. Set up the formula.
- d. Substitute and solve.
- e. How many degrees is the marked angle? _____°



Use sine, cosine, and tangent to find the angle measurement to the nearest whole degree.



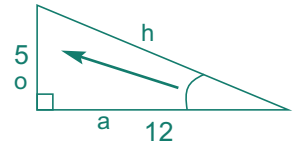
2. Bill wants to put solar panels on the house roof for heating water. Bill needs to know



how many degrees are in the lower angle of the roof in order to make his panel mounting brackets. The roof rises 5 inches for every 12 horizontal inches.

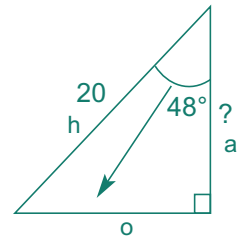
Lesson 12

- Known information: The lengths of the _____ and the _____ sides.
- Desired information: The _____ for the marked angle.
- Set up the formula.
- Substitute and solve.
- How many degrees are in the lower angle of the roof? _____°



Use the chart on page 67 to find the missing length to the nearest tenth.

- Known information: The length of the _____ and the number of degrees for the marked angle.
 - Desired information: What is the length of the _____ side?
 - The cosine for 48° from the chart is _____
 - Set up the formula.
 - Substitute and solve.
 - What is the length of the adjacent side? _____



We Remember

Complete the sentence.

- Reverse the inequality sign if you _____ or _____ both sides by a negative.

Solve.

5. a. $\frac{w}{-4} > 6$

b. $\frac{x}{3} - 6 > -3$

c. $-y - 1 < -2$

d. $4z - 5 > 7$



Find the interest compounded yearly on \$600 savings at 6% interest for 4 years.

6. Year 1 principal a. _____ interest b. _____
 7. Year 2 principal a. _____ interest b. _____
 8. Year 3 principal a. _____ interest b. _____
 9. Year 4 principal a. _____ interest b. _____
 10. Ending balance a. _____ Total interest b. _____

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11. The formula for finding the area of a trapezoid is _____.
12. Parts of an expression separated by positive or negative signs are called _____.
13. a. The repeating decimal for $\frac{1}{6}$ is _____. b. The decimal for $\frac{7}{8}$ is _____.
14. a. $\frac{\text{opposite side}}{\text{hypotenuse}}$ is the _____. b. $\frac{\text{opposite side}}{\text{adjacent side}}$ is the _____.
 15. a. 1 gallon = _____ fluid ounces b. 1 pint = _____ fluid ounces

Simplify and solve.

16. a. $24y + 10(-2) = -3y - (-6 + 8)$ b. $-8(x - 6) = 3(-3x - 2)$ c. $2x^2 + 1 = 99$



Complete the table showing the number of different ways Westside Equipment resold used equipment they had bought. Use this information to construct a rectangle graph.

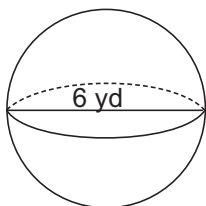
△ 17.

How Used Equipment Was Sold	Pieces Sold	Percent
As operational equipment	12	
As a unit for parts	7	
As parts only	16	
Total		

Lessons 12, 13



Find the volume of the sphere. Use 3.14 for pi. Round to the nearest whole yd^3 . $V = \frac{4}{3}\pi r^3$



18. _____

Lesson

13



Volume of an Irregular Solid

You have learned to find the volume of rectangular and triangular prisms and cylinders using the formula $V=Bh$. You have also learned to find the volume of a pyramid or cone using the formula $V=\frac{1}{3}Bh$, and the volume of a sphere using $V=\frac{4}{3}\pi r^3$.

Many familiar objects combine two or more of these regular shapes into one irregular solid. Grain bins may combine a cylinder and a cone or a rectangular prism and a pyramid. Buildings often combine a rectangular prism for the lower part of the building with a triangular prism for the roof. Other objects may be a little more complex. A propane tank may be a cylinder with a hemisphere (half a sphere) on each end. A house roof with no gable combines a triangular prism with half a pyramid at each end.

Example 1: The grain bin has a total height of 12 feet from top to point. The cylinder part is 5 feet high. The radius of both the cone and the cylinder is 3 ft. How many bushels will this grain bin hold?

- Find the volume of the cone.
- Find the volume of the cylinder.
- Find the total volume.
- Find the number of bushels.

a.

$$\begin{aligned} V &= \frac{1}{3}Bh \\ V &= \frac{1}{3}(\pi r^2)h \\ V &= \frac{1}{3}(3.14 \cdot 3 \cdot 3)7 \\ V &= \frac{1}{3}(28.26 \cdot 7) \\ V &= \frac{1}{3} \cdot 197.82 \\ V &= 65.94 \end{aligned}$$

b.

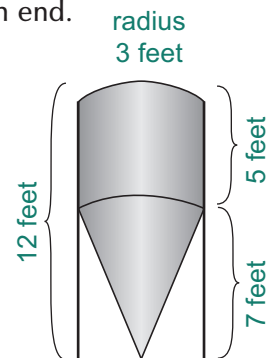
$$\begin{aligned} V &= Bh \\ V &= (\pi r^2)h \\ V &= (3.14 \cdot 3 \cdot 3)5 \\ V &= 28.26 \cdot 5 \\ V &= 141.3 \end{aligned}$$

c.

$$\begin{aligned} &\text{Upper part} + \text{lower part} \\ &141.3 + 65.94 = 207.24 \text{ ft}^3 \end{aligned}$$

d.

$$\begin{aligned} &1 \text{ ft}^3 \approx 0.8 \text{ bushels} \\ &207.24 \text{ ft}^3 \cdot 0.8 = 165.792 \text{ bushels} \end{aligned}$$



The grain bin holds about 165.792 (or almost 166) bushels.